

Global Stock Market Interdependence or Contagion? A Copula Approach

Yanyan Yang

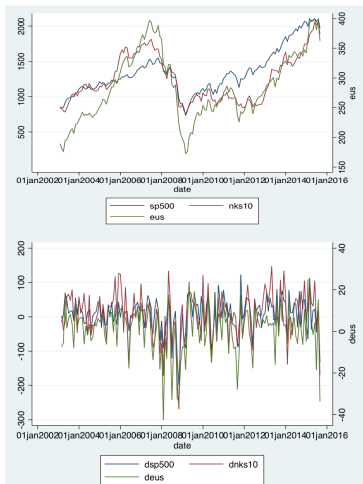
Claremont Graduate University

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- 1 A Survey of Measures: From Pearson Correlation to Copula
- 2 Methodology
- 3 Empirical Application and Results
- 4 Conclusion and Future Research

Background

- The correlation among SP500 Euro Stoxx 50 and Nethree stock markets is above 0.75. The correlation among the changes of these three stock markets is above 0.65.



Financial Contagion

1. the wake-up-call contagion
2. the shift contagion through financial channel or trade channel
3. the pure contagion: non-fundamental contagion

The major interest is the "pure" contagion due to the spillover of the market sentiment.

Research Questions

Q1

Is there a contagion-a significant increase of interdependence during crisis?

Q2

Is the interdependence between stock market returns tail dependent?

Q3

Can sentiment explain this global stock market interdependence?

From Pearson Correlation to Copula

Standard tests of contagion such as Forbes and Rigobon(2002) examine if cross-market correlations in stock market returns increase during a period of crisis.

Simple Correlation: Pearson Correlation

Rank Correlation: Kendalls τ Correlation and Spearmans ρ Correlation

Dynamic Correlation: Multivariate GARCH Model

Copula

Rank Correlation

Mathematically, any pair of observations (x_i, y_i) and (x_j, y_j) , where x_i, y_i, x_j, y_j are said to be concordant if the ranks for both elements agree: that is, if both $x_i > x_j$ and $y_i > y_j$ or if both $x_i < x_j$ and $y_i < y_j$. They are said to be discordant, if $x_i > x_j$ and $y_i < y_j$ or if $x_i < x_j$ and $y_i > y_j$. If $x_i = x_j$ or $y_i = y_j$, the pair is neither concordant nor discordant.

Definition of Kendall's tau of two variables X and Y is

$$\rho_\tau(X, Y) = 4 \int_0^1 \int_0^1 C(u, z) dC(u, z) - 1, \quad (2)$$

Definition of Spearman's rho of two variables X and Y is given by

$$\rho_s(X, Y) = 12 \int_0^1 \int_0^1 C(u, z) du dz - 3, \quad (3)$$

Intuitively a pair of random variables is concordant whenever large values of one variable are associated with large values of the other variable.

What is a Copula?

- The concept of copula was introduced by Sklar (1959), and has been applied in finance almost two decades later. The advantage of copula approach is that appropriate marginal distributions for the components of a multivariate system can be selected freely without the strict assumption of normal distribution.
- It is a function.
- It links marginal distributions to form a joint distribution

Why Copula?

Non-Normality, Fat Tail, Asymmetry

Shape of Distribution

Sample Size

Outliers

Non-linearity

Copula Functions

Gaussian Copula

$$C_{\rho}(u, z) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(z)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left\{\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right\} dx dy, \quad (4)$$

Student t Copula

$$C_{\rho, v}(u, z) = \int_{-\infty}^{t_u^{-1}(u)} \int_{-\infty}^{t_v^{-1}(z)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \left\{1 + \frac{x^2 - 2\rho xy + y^2}{v(1-\rho^2)}\right\}^{-(v+2)/2} ds dt, \quad (5)$$

Clayton Copula

$$C_{\delta}(u, z) = (u^{-\delta} + z^{-\delta} - 1)^{-1/\delta}, \quad (6)$$

Gumbel Copula

$$C_{\delta}(u, z) = \exp\left(-\left[(-\log u)^{\delta} + (-\log z)^{\delta}\right]^{\frac{1}{\delta}}\right), \quad (7)$$

Frank Copula

$$C_{\delta}(u, z) = \exp\left(-\left[(-\log u)^{\delta} + (-\log z)^{\delta}\right]^{\frac{1}{\delta}}\right), \quad (8)$$

Investor Sentiment Measurement

- To measure the sentiment, I am using a new survey-based data set. It separates the institutional investors from individual investors. This weekly data set surveys more than 800 investors including over 20 percent institutional investors from July 2001 to August 2015
- It does not only include the percentage of investors with bullish and bearish beliefs concerning on different equity markets, but also includes the expectations from the same subject in a single survey regarding the US, European and Japanese economies.
- Stock Market Indices Sentiment: SP500, Euro-Stoxx 50, NOKKKEI 225
- Economies Sentiment: the US, Europe, Japan

Example of Survey Questions

* 4. S&P 500

	bullish	neutral	bearish	keine Meinung no opinion
kurzfristig (1 Monat) short term (1 month)	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
mittelfristig (6 Monate) medium term (6 Monate)	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

* 5. Nasdaq Composite ([Nasdaq](#))

	bullish	neutral	bearish	keine Meinung no opinion
kurzfristig (1 Monat) short term (1 month)	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
mittelfristig (6 Monate) medium term (6 Monate)	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

* 6. Nikkei 225

	bullish	neutral	bearish	keine Meinung no opinion
kurzfristig (1 Monat) short term (1 month)	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
mittelfristig (6 Monate) medium term (6 Monate)	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

* 7. CSI 300 Index ([Blue Chips China](#))

	bullish	neutral	bearish	keine Meinung no opinion
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Marginal Distribution

To deal with the time-varying volatility, I apply GARCH(1, 1) Model based on the previous literature Engel (1994)

The marginal distribution for the stock market index return is assumed to be characterized by AR(1)-GARCH(1,1) specification.

Table 1: Summary Statistics

Descriptive statistics of weekly three major index returns US, Europe and Japan from Jan 2004 to April 2016.

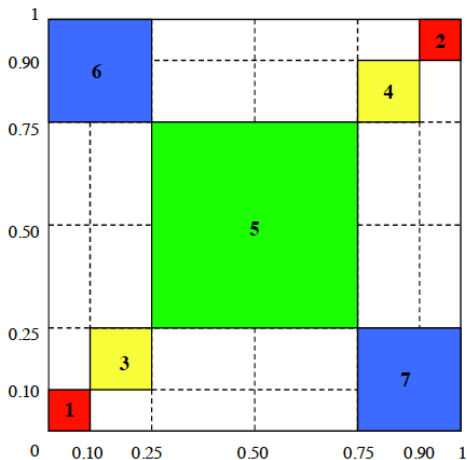
Summary of Statistics														
Index	N	mean	sd	min	max	Var	skewness	kurtosis	p1	p5	p10	p90	p95	p99
SP500	644	0.0013	0.024	-0.18	0.12	0.0006	-0.613	10.57	-0.068	-0.038	-0.025	0.026	0.033	0.062
European 50	644	0.0006	0.0299	-0.22	0.122	0.0009	-0.819	8.949	-0.08	-0.045	-0.035	0.032	0.043	0.074
NIKKEI 500	644	0.0012	0.0305	-0.24	0.121	0.0009	-1.009	9.957	-0.073	-0.05	-0.033	0.035	0.044	0.068


Correlation Matrix


Table 2: Simple Correlation & Rank Correlation

Index	SP500	European 50	NIKKEI 500
Pearson Correlation			
SP500			
European 50	0.835		
NIKKEI 500	0.632	0.656	
Kendall's τ Correlation			
SP500			
European 50	0.613		
NIKKEI 500	0.382	0.417	
Spearman's ρ Correlation			
SP500			
European 50	0.799		
NIKKEI 500	0.545	0.587	


Hit Regions



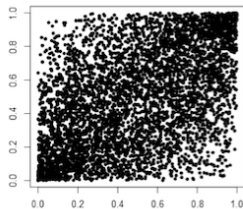
 *Extreme up/down days*

 *Moderate up/down days*

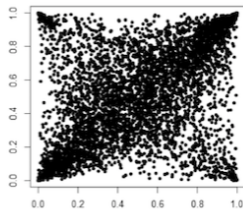
 *Median days*

 *Asymmetric days*

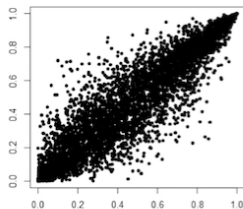
Simulations of Different Copula



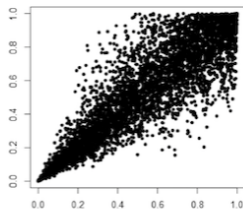
Bivariate Gaussian copula with $\rho = 0.5$



Bivariate Student-t copula with $\rho = 0.5$ and dof = 1



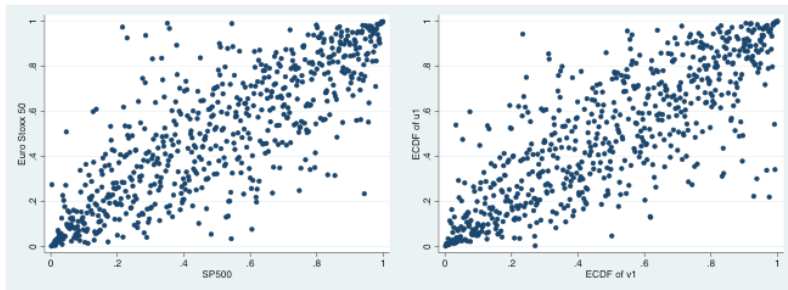
Bivariate Gumbel copula with $\alpha = 4$



Bivariate Clayton copula with $\alpha = 5$

Real Data: Unconditional Transform v.s. Conditional Transform

Figure 4 Scatter Graphs of Probability Integral Transformed Stock Market Indices Returns



Left: SP500 NIKKEI225 Right: Euro Stoxx 50 NIKKEI225

Figure 5 Left: SP500 & NIKKEI225

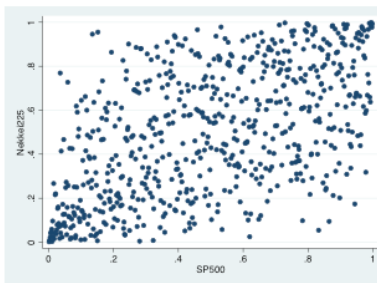
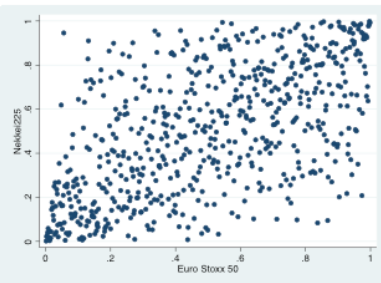


Figure 5 Right: Euro Stoxx 50 & NIKKEI225



Sensitivity Tests

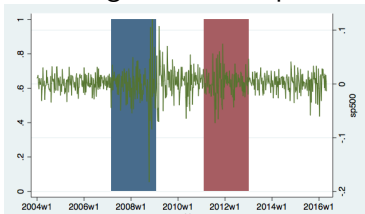
Example 1: Recession v.s. Normal Periods

I am using the recession indicators from Fed St, Louis to define the recession periods

Example 2: Stable Periods v.s. Turmoil Period

Example 3: US Financial Crisis v.s. Euro Crisis

It is immediately apparent that adjusting for heteroskedasticity has a significant impact on estimated cross-market correlations and the resulting tests for contagion. The unconditional correlation is substantially smaller than the conditional correlation during the turmoil period and is slightly



Copula Tail-dependence Results: Tail-dependence copulas provide best fitting.

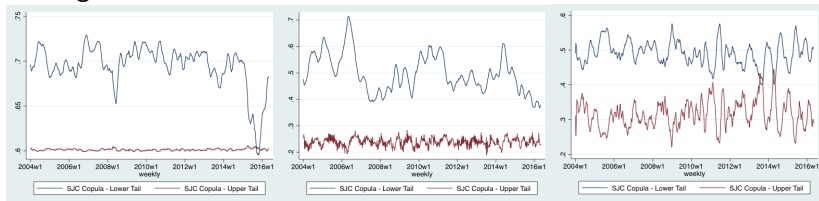
SP500 & European 500			
LL	Lower Tail	Upper Tail	
9	-366.166	0.6959	0.6024
7	-361.8708	0.6862	0
1	-347.3327	0	0
4	-343.1871	0	0
6	-335.388	0	0.6715
5	-327.8922	0	0
2	-314.1655	0.7391	0
3	-267.1089	0	0.7047
8	-154.1631	0.109	0.109

SP500 & NIKKEI 500			
LL	Lower Tail	Upper Tail	
7	-361.8708	0.6862	0
9	-139.2411	0.4755	0.2536
8	-133.7148	0.0084	0.0084
1	-133.4138	0	0
2	-125.659	0.506	0
4	-113.4247	0	0
5	-112.8448	0	0
6	-109.6251	0	0.4324
3	-84.7202	0	0.4134

European 500 & NIKKEI 500			
LL	Lower Tail	Upper Tail	
9	-155.3872	0.4916	0.3112
8	-154.1631	0.109	0.109
7	-153.048	0.4915	0
1	-152.068	0	0
4	-141.9754	0	0
2	-138.1665	0.5306	0
5	-137.4178	0	0
6	-131.1288	0	0.4696
3	-101.0435	0	0.4607

Time-Varying Copula Results

These parameters can't be interpreted as correlation since they enter into the logistic transformation.



My Research Philosophy: Poetry is when you make new things familiar and familiar things new. Same as research.

- Short-term
 - High Dimensional Time-Varying Copula
 - The other possible extension could be researching on emerging markets before and after the crises.
- Long-term
 - I try to introduce behavioral finance into the standard international finance framework, specifically highlighting the effect of investor sentiment.
 - I would like to use a variety of methodologies to answer questions on these topics including standard theoretical analysis, econometrics, and non-parametric methodology and laboratory experiments.

Thank you very much!
Questions?